WinMaC 2017

Guts Round

Set 1

1. **(3)** Ryan has a bad taste in music, his friend Jeffrey came over to his house and listened to some of it. If it took Jeffrey 70 minutes before he left Ryans house due to the annoying music, and each song averages 2 minutes, how many songs did Ryan play to Jeffrey if he didnt stop playing music?

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2. **(3)** How many different positive intergers less than 17 share a common factor with 17?

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3. **(3)** What is $1^2 + 2^2 + ... + 5^2$?

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Team: ________________

Set 2

4. **(4)** An Earth day consists of about 24 hours, and a day on Planet WinMaC consists of 12 hours. When a week passes on the Earth, how many days pass on planet WinMaC?

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5. **(4)** Other than 1, what is the smallest natural number that is both a perfect square and a perfect cube? (For example, 8 is a perfect cube because it is 2^3)

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6. **(4)** Ten bees compete in a spelling competition. Suppose there are B bees in a given round. Then, if B is even, $\frac{B}{2}$ bees advance to the next round. If B is odd, then $\frac{(B+1)}{2}$ bees advance. How many rounds will it take to determine a single bee that wins?

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Team: ________________

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Set 3

7. **(5)** The probability P that dinosaurs once lived is equal to $\frac{P+1}{6} + \frac{2}{3}$. What is the probability that dinosaurs once lived?

8. **(5)** Will and Jason are racing around a circular track. Will can run one lap around the track in 15 seconds while Jason can run one lap around the track in 30 seconds. If they start at the same place, and run in opposite directions, how many second will meet each other for the first time?

9. **(5)** Jason is thinking of a positive integer less than 150. He gives it to Will, who continues to divide the integer by 2 until he receives a fraction. He is able to divide by 2 five times, such that the result is always an integer. If the integer Jason thought of is a multiple of three, what is his integer?

Team: ________________

Set 4

10. **(6)** The hypotenuse of a right triangle is $2x + 9$, and the legs of the right triangle are $2x + 1$ and 12, what is the value of x?

11. **(6)** At the school store, Jared must buy pencils, erasers, markers, and notebooks. He needs at least two pencils, and at least one eraser, marker, and notebook. However, he can only buy at most 7 items from the store. In how many ways can he buy his items?

12. **(6)** If $n - 10$ has 5 factors and $n + 10$ has 9 factors, then how many factors does n have?

Team: ________________

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Set 5

13. **(7)** 3 distinct positive integers have a common LCM of 120, what is the positive difference of the greatest sum of the 3 numbers and the smallest sum of the 3 numbers?

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14. **(7)** The weather report is giving a 75% chance for one foot of snow and 25% for 8 inches of snow for tomorrows forecast. If the local public school cancel school 50% of the times when there is 8 inches of snow and 80% of the times where there is a foot of snow, what is the possibility that there will be a snow day tomorrow? Write your answer as a fraction.

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15. **(7)** There is a square inside of a 6×6 grid. If all vertices of the square are lattice points and lie on the border of the grid, what is the largest possible area of the smaller square?

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Team: ____________________

Set 6

16. **(8)** Jonathan was feeling bored one day, so he went to the local slot machine. The slot machine had the numbers 1 through 5 on 3 slots. Jonathan will win money if the product of the 3 numbers is a perfect square. What are the odds of him winning?

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17. **(8)** After the first semester, $\frac{2}{7}$ of the Algebra II class dropped down to the Algebra I class, and $\frac{1}{3}$ of the Algebra I class moved up to the Algebra II class. If everyone at this school attends exactly one of these two math courses, and the number of students in each class after the first semester was equal, what is the lowest possible number of students?

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18. **(8)** Let $a_1, a_2, a_3, \cdots, a_{15}$ be reals such that the system of equations

$$a_1 + a_2 = 1,$$

$$a_2 + a_3 = 2,$$

$$\cdots$$

$$a_{14} + a_{15} = 14,$$

$$a_{15} + a_1 = 15$$

is satisfied. What is the value of a_{15}?

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Team: ____________________

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Set 7

19. **(9)** Given a system $ax + by = 2$ and $cx + dy = 6$, where the domain of a, b, c, and d are integers belonging to [1, 5]. How many ways are there to make this system have 0 solutions?

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20. **(9)** Let a and b be positive integers such that $a + b = 21$ and $a^2 + b^2 = 415$. What is $a^3 + b^3$?

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21. **(9)** In $\triangle ABC$, let points D and E be on sides $\overline{BC}$ and $\overline{AC}$ respectively, such that $\overline{AD} \perp \overline{BC}$ and $\overline{BE} \perp \overline{AC}$. Let $\overline{AD}$ intersect $\overline{BE}$ at point H, and $AH = BH$. Given that $\angle ACB = 72°$, compute the number of degrees in $\angle ABC$.

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Team: _______________

Set 8

22. **(10)** Given triangle $\triangle ABC$, $m\angle A = 45°$, $m\angle B = 45°$, and $m\angle C = 90°$. Let point D lie on side $\overline{BC}$ such that $m\angle DAC = 30°$. Furthermore, let point E lie on side $\overline{AC}$, and point F lie on $\overline{AB}$, such that $m\angle BDF = 75°$, and $m\angle CDE = 15°$. if $AD = \sqrt{1254}$, find the value of $2[AFE] + [DFE]$, where the brackets denote area.

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23. **(10)** Jason and Robert play a boring game. First, Jason chooses a polynomial $P(x)$ of degree 2017. That is, $P(x) = a_{2017}x^{2017} + a_{2016}x^{2016} + \cdots + a_2x^2 + a_1x + a_0$ for reals $a_0, a_1, \cdots, a_{2017}$. Robert must choose a real value of x and Jason will tell him the value of $P(x)$. Then, they repeat the process until Robert has determined the value of $a_1 + a_3 + a_5 + \cdots + a_{2017}$. If Robert plays with an optimal strategy, what is the minimum number of x values Robert needs to choose to succeed, with disregard to what polynomial Jason chooses?

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24. **(10)** $ABCD$ is a square with side lengths 5. E is a point outside the square such that $CE = 5$, $BE = 8$, and $DE = 7\sqrt{2}$. A circle is inscribed in $\triangle BCE$ with center O. If $\overline{DE}$ does not intersect $\overline{BC}$ and $\overline{BE}$ does not intersect $\overline{CD}$, then find the area of $\triangle BDO$.

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