
Set 1

1. (4) The measures of the angles of a triangle are $(2x + 17)^\circ$, $(x + 43)^\circ$, and $(5x - 48)^\circ$. Find the measure of the largest angle of this triangle.

2. (4) The centered hexagonal numbers are 1, 7, 19, 37, 61, 91, etc., where the n th hexagonal number is larger than the $(n - 1)$ th hexagonal number by $6(n - 1)$. Find the sum of the first 10 hexagonal numbers.

3. (4) I am on a bullet train that is travelling at 300km/h. I start at the back of the train and run to the front of the train and back at 10km/h. The train is 0.1km long. During the time it takes me to run 10 laps, what will my average speed be (relative to the ground)?

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Set 2

1. (4) Gabe can run at a speed of 400 meters every 60 seconds. However, for each successive 400 meters he runs, his time to run the next 400 meters increases by 1 second. How long will it take him to run 1600 meters, in MINUTES? Express your answer in decimal form.

2. (4) What is $\frac{((2^2)^2)^2}{((2^2)(2^2))^2}$?

3. (4) How many milliliters of water does a cylindrical tank hold if it is 50 centimeters deep with a diameter of 20 centimeters? Express your answer in terms of π . (Hint: one milliliter is equal to one cubic centimeter.)

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1. (4) Mr. Mac turns to a random direction (North, South, East, or West, each equally likely to be chosen) and takes 1 step. He does this process a total of four times. What is the probability that he ends up in the exact same place he started?

2. (4) My study and focus music playlist has 20 songs: 5 are classical, 3 are country, 7 are pop, 1 is rock, and 4 are jazz. If I hit the shuffle button, my songs will be arranged in random order. The probability that the first 9 songs are classical, jazz, classical, jazz, classical, jazz, classical, jazz, and classical, in that order, can be expressed as the fraction

$$\frac{1}{2^a * 3^b * 5^c * 7^d * 11^e * 13^f * 17^g * 19^h}$$

where $a, b, c, d, e, f, g,$ and h are nonnegative integers that are not necessarily distinct. What is the value of $a + b + c + d + e + f + g + h$?

3. (4) If $a^2 + ab - 3a = 0$ and $\frac{a}{2} = \frac{4}{b^2} - \frac{a^2}{2b}$, find $a^3 + b^3$.

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Set 4

1. (4) $gcd(60, x) = 6, lcm(60, x) = 180$. Find x .

2. (4) There are 50 students in a class. 33 like math, 33 like music, and 33 like recess. Each student likes at least one activity. Of those who like math, 17 like music, and 17 like recess. Of those who like recess, 17 like music. How many people like all three activities?

3. (4) The sum of the factors of a number, including itself, is 36. Find this number.

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1. (4) In base 10, the last two digits of a number are 67. In base 3, the last two digits of the same number are 21. If it is a three digit number in base 10, find this number.

2. (4) If a is positive, and $a^2 = (2 * 5 - 1) * (5 * 13 - 1) * (13 * 34 - 1)$, find a .

3. (4) What is $1 + (2 + 1)(2 - 1) + (4 - 2)(4 + 2) + (8 - 4)(8 + 4) + (16 - 8)(16 + 8) + (32 - 16)(32 + 16)$?

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Set 6

1. (4) If there are 10 identical green blocks, and 9 distinct red blocks, how many ways are there to arrange them in a line so that no two red blocks are next to each other?

2. (4) Given $q^2 + r^2 = 2$, and $s^2 + t^2 = 4$, find $(qs + rt)^2 + (qt - rs)^2$.

3. (4) A number is called "three-fifths perfect" if the sum of its factors **not including** the number itself is three fifths of the original number. Find the smallest positive "three-fifths perfect" number.

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Set 7

1. (4) A sequence T_n is defined by $T_1 = 1$, $T_2 = 2$, and $T_{n-1} * T_{n+1} = (T_n)^2 - 1$. Find T_{2023} .

2. (4) There are four 2-digit numbers that produce a number with the last 2 digits "01" when squared. Find their sum.

3. (4) Find $\sqrt{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3}$

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Set 8

1. (4) Given:

$$a + b + c + d + e + f = 1$$

$$a + 2b + 4c + 8d + 16e = 32$$

$$a + 3b + 9c + 27d + 81e = 243$$

$$a + 4b + 16c + 64d + 256e = 1024$$

$$a + 5b + 25c + 125d + 625e = 3125$$

$$\text{Find } a + 6b + 36c + 216d + 1296e$$

2. (4) Find the sum $(0 \cdot 1 + 1 \cdot 2) + (1 \cdot 2 + 2 \cdot 3) + (2 \cdot 3 + 3 \cdot 4) + (3 \cdot 4 + 4 \cdot 5) + \dots + (19 \cdot 20 + 20 \cdot 21)$.

3. (4) Find $\sqrt[3]{18 + \sqrt{325}} + \sqrt[3]{18 - \sqrt{325}}$.

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3. (4) Find $\sqrt[3]{18 + \sqrt{325}} + \sqrt[3]{18 - \sqrt{325}}$.
